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Contributions to Jet Noise from Instability Waves and their Interactions: From Theory to Modelling

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Abstract

Experiment and direct numerical simulation provide compelling evidence that nonlinear wave interactions play a significant role in the generation of noise in subsonic jets. A simple 'difference mode' approach captures the essential mechanism, which is a much stronger source of large structure sound, particularly at low Mach numbers, than the alternative of direct linear conversion. Significant features of the low frequency end of the jet noise spectrum are predicted correctly, including the shift of the spectral peak to lower frequencies, the directivity pattern and the effect of Mach number. This mechanism distinguishes large structure sound from that associated with the breakdown to small scales. For practical calculations of jet noise a hybrid approach is proposed in which the Reynolds-Averaged Navier-Stokes equations are used to compute a base flow solution and then either a truncated Navier-Stokes approach, reducing to linearised Euler in the far field, or a Parabolised Stability Equations solver coupled to a linearised Euler solver are used to calculate sound sources and propagation. This approach potentially provides a coupling between flow control on the jet nozzle and the radiated sound.

Keywords: jet noise, direct numerical simulation, nonlinear interaction

1. Introduction

In this paper we discuss the feasibility of predicting aspects of jet noise from knowledge of two aspects of the jet flow: (i) the time-averaged base flow, such as may be obtained from solution of the Reynolds-averaged Navier-Stokes equations, and (ii) the instability characteristics, which may be obtained from the truncated Navier-Stokes equations, or from a number of further simplified stability theories. Since the former is straightforward, attention is focused on the second aspect and we will use solutions of the full Navier-Stokes equations to see what can and cannot be expected of such a model.

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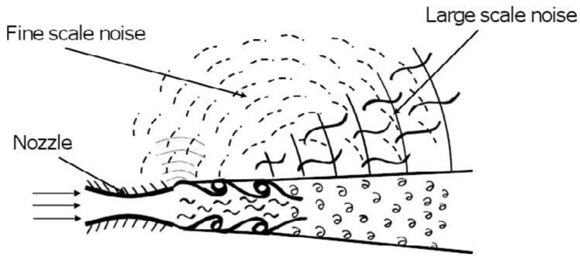

Figure 1: Schematic of jet structure and radiated sound.

Most studies of jet noise accept that there is a connection between wavepackets (the detailed structure of which is left unspecified) inside the jet column and some part of the radiated sound. This is often phrased in terms of 'large structure' sound versus other sorts of sound, which are labelled as 'small-scale' in origin. Figure 1 shows how the various sources co-exist in a jet. This decomposition is implied in previous work [1, 2] that empirically attributes these two types of sound with different spectral content. At this level there is no insight into physical mechanisms, nor is any prediction scheme proposed. Possibilities for the latter appear when we make a further step and connect large scale structure with instability waves, for which there is a large literature. This is clearly on the right track for jets in which the instability waves have a supersonic phase speed and there can be direct Mach wave radiation [3, 4, 5].

For subsonic phase speeds one can still make the connection between instability waves and sound if one recognises that, due to the jet growth, waves at a particular frequency have an amplitude envelope that grows and decays as the wave progresses downstream, e.g. [6]. The simple argument is then made that a Fourier decomposed signal (transforming in the downstream direction) has some components with an underlying supersonic phase speed, and these can radiate. This process has been emphasized several times (e.g. [5, 6]) and is certainly valid. As we shall see, the problems are (i) that it underestimates the size of the sound radiation and (ii) has the peak sound radiation at the wrong frequency.

The latter issue is resolved if one considers interactions of instability waves, since waves can be strongly amplified within the jet (giving the hydrodynamic spectrum) but radiate at the interaction frequencies (with a different spectrum). There is an emerging body of evidence available from numerical simulations (e.g. [7, 8]) that this process is important, and also convincing evidence from older (but little-known) experiments [9] that the mechanism works efficiently even when the underlying jet is fully turbulent. In the following sections we review the evidence and make the case that at least part of the spectrum and directivity of turbulent jet noise is amenable to prediction with a combination of current numerical tools.

2. Linear mechanism of sound generation from hydrodynamic instability waves

Before discussing the more important nonlinear mechanisms, we first provide a brief review of the accepted linear mechanism. Such a mechanism of sound production appears

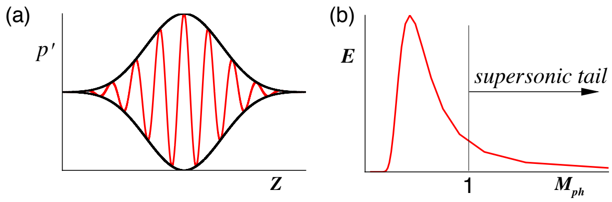

Figure 2: Sketch of the wave packet model of sound radiation: (a) wavepacket at frequency f and its envelope; (b) energy as a function of phase Mach number arising from taking a Fourier transform of (a) in the streamwise direction.

to be dominant for high supersonic jets where structures (linear eigenmodes or turbulent eddies) propagate with supersonic speeds. In this case the classic wavy wall solution (see e.g. [10]) is applicable and the sound is directly connected to jet structures. Some examples confirming the role of instability waves in sound radiation from supersonic jets are given in [3, 4, 11]. This mechanism does not as it stands apply to subsonic jets. However once we recognise that instability waves (and by analogy turbulent eddy structures) follow a lifetime of growth, saturation and decay, we can adopt a wave packet model, sketched on figure 2. Because of the jet spreading, an instability wave at frequency f undergoes growth and decay while propagating in the streamwise direction (Fig. 2a). As a result, a wavenumber spectrum, corresponding to the signal plotted in Fig. 2a, is broadband. Therefore some part of the spectra contains waves with supersonic phase velocity (Fig. 2b) and these can radiate. Sound waves radiated by such a mechanism are commonly identified as Mach waves, e.g. [5]. Most studies of sound mechanisms stop at this point.

3. Evidence for nonlinear interactions as sound sources in subsonic jets

It is important to state at the outset that the nonlinear interactions we have in mind cover a wide spectrum of interactions. We consider a discrete set of frequencies $w_i = n\Delta\omega$ for $n = 1 \dots N$, at M different azimuthal modes (azimuthal wavenumbers $m = 0, \pm 1 ... L$, where $M = 2L + 1$) typically with the range covering all waves exhibiting a significant degree of instability in the jet.

Quadratic nonlinear interactions then lead to M^2N^2 possible interactions. Calculations along these lines will be discussed later as they are more representative of jets developing from the broadband background noise that is always present to some degree in laboratory experiments. However, to assess the basic mechanism it is useful to consider the interaction of any two waves.

3.1. Experiment

An experimental study by Ronneberger & Ackermann [9] appears to be the first publication that explicitly examined the role of nonlinear interactions in jet noise. It has disappeared under the radar of recent jet noise literature so we recall the essential elements here. The experiments were conducted for a round jet at moderate subsonic Mach numbers (equal to 0.37 and 0.44 for two separate experiments). In the first experiment the jet was forced at two frequencies $St_1 = 0.5$ and $St_2 = 0.3$ and in the second at $St_1 = 0.7$ and $St_2 = 0.5$, where St is the Strouhal number based on the jet diameter and centreline velocity. The forcing was weak, in the sense that the resulting pressure amplitudes were not too dissimilar to the unexcited jet, which was also measured. The near field showed that the difference mode, $\Delta St = 0.2$, differed between the two cases, consistent with a weakly nonlinear mechanism rather than a linear response to natural forcing at $St = 0.2$. In the acoustic far field (more than 100 diameters from the jet) the sound pressure level measured for the second frequency combination was more than 10 dB higher than that obtained with loudspeaker forcing at $St = 0.2$ applied in the settling chamber and more than 20dB higher when the forcing was applied with a quieter technique. The measured sound at the difference frequency was strongly directed downstream, decreasing by 20 dB from 20° to 70° (angles measured relative to the jet axis). The study also states that radiation at the difference frequency dominates over that of other nonlinear interactions.

Quadratic nonlinearity was also apparent in [12], for forced vortex pairing in a round jet, where the sound intensity was observed to vary as the fourth power of the near-field mode saturation amplitude, whereas a second power would be expected for a linear relationship of the type assumed in [13]. Experimental results of Stromberg et al. [14] for a subsonic low Reynolds number jet (Re=3600, M = 0.9) also indicate that a nonlinear mechanism involving the dominant St=0.44 instability waves is responsible for a significant portion of the peak noise generated from the jet (around St=0.22). Other works have mentioned nonlinear mechanisms but do not provided quite so explicit a demonstration of the effect as [9].

3.2. Direct numerical simulations

The investigation of plane jet noise based on the impulse response of subsonic plane jets reported in [7, 15] was not originally a study of nonlinear interactions but these emerged as an explanation for the phenomena seen in the numerical simulations. Here we take some examples from more recent numerical simulations [16, 17] that consider round jets. These simulations were set up so that linear and weakly nonlinear phenomena could be observed in detail. A representative base flow (a jet with initially a top hat velocity profile evolving downstream into a gaussian velocity profile) was fixed so that it would be maintained in the absence of upstream disturbances. The profiles are convectively unstable and act as a filter/amplifier of disturbances that are specified at the inflow plane. At low forcing amplitude the response is completely linear and the disturbance evolution can be used as a validation of linear stability theories, while at higher amplitudes the response is nonlinear and the same configuration can be used to assess nonlinear stability theories [18].

Here we focus on the characteristic response observed in the simulations. Figure 3 (a-c) shows the response of the flow at three different Mach numbers to forcing at a single frequency $\omega = 1.2$. The figures show contours of dilatation rate. The flow is unstable at this frequency and along the jet axis one can see the development of a recognisable wave packet, with maximum amplitude at $z/D \approx 7$, where D is the jet diameter. The amplitude of the inflow disturbances is 10^{-3}. At the disturbance maximum this amplitude has grown to ≈ 0.1. Note however that there is no breakdown to turbulence in this case, so only large structures are seen. At the higher Mach numbers it is clear that

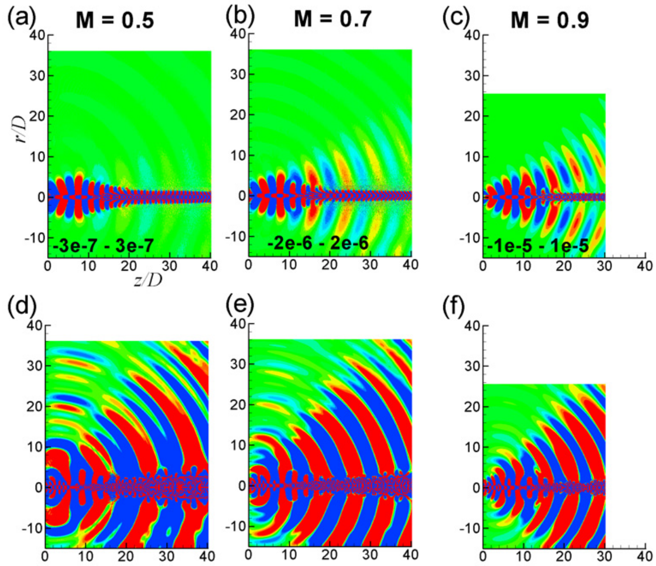

Figure 3: Dilatation rate contours: (a), (b) and (c) single-frequency axisymmetric forcing at $\omega = 1.2$, (d), (e) and (f) 2-frequency axisymmetric forcing at $\omega_1 = 2.2$ & $\omega_2 = 3.4$. Contour values vary with Mach number but are the same for each of the Mach numbers, i.e. for (a) & (d), (b) & (e) and (c) & (f).

some sound is radiated from this single frequency case, corresponding to the classical wavepacket argument discussed in section 2.

Figure 3 (d-f) shows the same configuration, but forced at two frequencies $\omega_1 = 2.2$ and $\omega_2 = 3.4$. Note that the inflow amplitudes are the same and the contour levels are the same as in the corresponding figures (a)-(c). Two effects are obvious. Firstly the sound is radiated with much greater amplitude and secondly the radiation frequency is the same as in the single frequency case i.e. the sound is radiated at the difference frequency $\Delta\omega = \omega_2 - \omega_1$. A number of factors determine the radiation amplitude. The excited waves in this case are at higher frequencies and are more unstable. This is a bigger effect than the fact that the source is now a multiple of two waves with amplitude $< O(1)$. Had we chosen very low amplitudes we would have seen evidence for direct radiation at ω_1 and ω_2 [8]. However, amplitudes 10^{-3} and above are certainly more representative of jet flows in practice, so the clear conclusion is that the 'difference mode' interaction mechanism is more efficient than the 'direct' radiation mechanism at subsonic Mach numbers. At higher Mach numbers [17] the direct mechanism is more efficient. An additional feature seen in Figure 3 is the wider angle of the radiated sound for the difference mode mechanism, corresponding more closely to the peak radiation

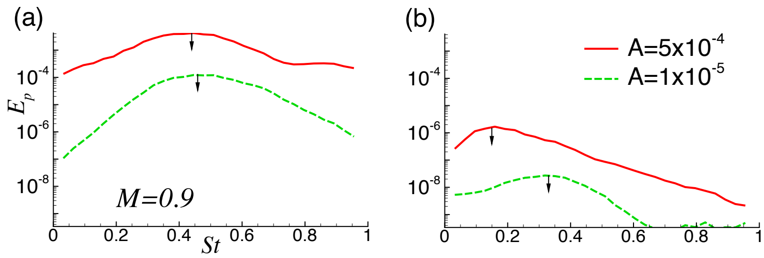

Figure 4: Spectra obtained for the flow forced over the frequency range $0.2 \leq \omega \leq 7$ at $m = 0$. (a) hydrodynamic spectra at $z/D = 5$ and $r/D = 0.5$; (b) acoustic spectra at $r/D = 40$ and $\theta = 30°$.

direction seen in subsonic jets.

4. Broadband sources, spectral shifts and small scales

For more realistic simulations we consider cases with inflow forcing over a broad range of frequencies $0.032 \leq St \leq 1.11$ (with 35 excited frequencies). Example spectra for the hydrodynamic field (figure 4(a) for two different amplitudes of the inflow forcing) show a spectral peak at $0.4 < St < 0.45$. In this case the flow was forced only with axisymmetric modes. The lower forcing amplitude gives essentially the linear response, while the higher amplitude (a factor of 50 higher) allows nonlinear effects. It can be seen that the Strouhal number inside the jet is unchanged, indicating that the dominant structures inside the jet are those that are most amplified by linear instability of the base flow.

When we consider the acoustic field for the same simulation we get a different picture. Figure 4(b) show the pressure spectrum calculated at $r/D = 40$ and $\theta = 30°$. As in the results shown in the previous section, the low amplitude case generate sound according to the direct linear mechanism. The spectral peak is at $St = 0.33$ which is reduced relative to the hydrodynamic field for reasons that will be discussed later. For the higher amplitude however the spectral peak is even further reduced to $St = 0.15$. The peak value of St depends on direction but we consistently observe values close to experiment, e.g. [2, 14]. This result is also valid for cases when the flow was forced with combinations of azimuthal modes. The ability of the interaction model to predict this significant spectral shift (more than a factor of two) between the spectral peak in the hydrodynamic field and the spectral peak in the acoustic field is one of the main successes of the approach.

When a moderate amplitude of forcing is used ($A = 5 \times 10^{-4}$ at azimuthal modes $m = 0, \pm1, \pm2$, giving 30625 possible quadratic interactions) the strong oblique mode of instability usually leads to breakdown to turbulence in the jet. Due to the prescribed base flow the location of the breakdown is typically downstream of the end of the jet potential core. This is illustrated in Figure 5 in which parts (a) and (b) show the sound field and enstrophy contours for the flow forced respectively at the axisymmetric $m = 0$ mode and at $m = 0, \pm1, \pm2$ modes. In the case of axisymmetric forcing no breakdown to turbulence is observed and the sound field includes only large structure sound, whereas

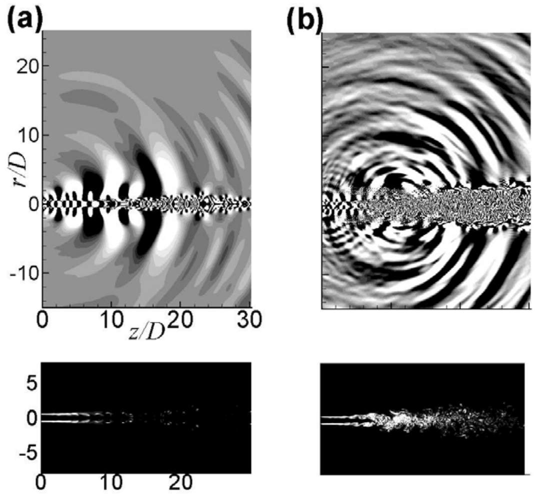

Figure 5: Dilatation rate contours and corresponding enstrophy contours. Flow was forced at frequency range $0.2 \leq \omega \leq 7$ and different combinations of the azimuthal modes (a) $m = 0$; (b)$m = 0,\ \pm 1,\ \pm 2$.

in the case of forcing by a combination of azimuthal modes we now also have sound due to breakdown to turbulence. The sound emanating from the turbulence breakdown (near $z/R = 10$ in Figure 5b) radiates more isotropically, with a significant contribution to the sideline noise.

5. Role of wave packets

We have mentioned wave packets in connection with the direct (same frequency) mechanism connecting instability waves to sound radiation. Of course, each of the non-linear interactions can also be identified as a wave packet, albeit one with a different envelope shape. It is therefore interesting to re-examine the results above from a wave-packet perspective.

It was already noted by [19] that sound radiation depended asymptotically on the shape of the wave packet envelope. The effectiveness of sound radiation was quantified for a model problem having growth, saturation and decay of wall modes [20], leading to three parameters: the Mach number, a dimensionless frequency and a dimensionless saturation parameter that captured the rate of transition from growth to decay (in other words the shape of the wave packet envelope near its peak amplitude). The model problem showed that sound radiation is more efficient for high Mach numbers, low frequencies and rapid saturation. The greater efficiency at low frequency explains the spectral shift seen for the linear response on figure 4(b). The spectral peak in the acoustic field was slightly lower ($St = 0.33$) compared to the spectral peak in the hydrodynamic field ($St = 0.45$).

The effect is more apparent at higher amplitude forcing where the low frequency end of the spectrum fills up, providing forcing of new wavepackets, which mainly arise at the difference frequencies of interacting strongly unstable linear modes. The increased efficiency of wavepacket radiation at low frequency combines with the more rapid saturation of the nonlinear interactions to more than compensate for the lower amplitude of the quadratic interactions (A^2 rather than A with $A < O(1)$).

6. Modelling approaches

In this final section of the paper we speculate on how the fundamental jet noise research outlined above might be turned into a more practical tool for jet noise prediction to complement current empirical approaches based on careful experimentation at both laboratory and full scale. The alternative of large-eddy simulation (LES) still needs some large increases in computer performance before it will be a routine tool to connect detailed nozzle design features (micro tabs, jets, plasmas etc) to the consequent sound radiation. LES grids have to span a wide range to cover the control device scale, the jet diameter scale and the radiation scale. Resolution of the far field sound also means fundamental limitations on the highest frequency that LES can reach. Thus it is of interest to examine alternatives and in particular whether the currently popular splitting of the acoustic spectra into that due to large scales and that due to 'fine scales', e.g. [1, 2], can be exploited to devise new prediction schemes. In the following section we consider hybrid techniques of various types.

6.1. Base flow from RANS

Any useful technique should be able to translate a mean flow change upstream in the nozzle into a downstream change in the jet. The obvious method to do this is RANS-based CFD, which despite its limitations is probably up to the task of, say, calculating the change in jet mean flow due to a vortex generator placed inside the nozzle. The change in mean flow due to chevron geometries is the kind of thing that RANS is (with care) able to do routinely. Thus we suppose that we can get from a design geometry to a RANS base flow. Compared to section 2 above this simply means replacing our prescribed analytic base flow with a RANS average flow field.

6.2. Perturbation evolution from truncated Navier-Stokes simulations

The simplest hybrid approach is to use RANS as a basis for calculations that are not dissimilar to those presented in Figure 3. In these calculations the amplitude is kept low so that only the weakly nonlinear mechanism is active. Such calculations can be cheap, as relatively few azimuthal modes need be retained. Resolution must be high in the radial and streamwise directions to resolve the source and sound fields, but in principle the calculations can be carried out well into the acoustic field. The numerical method needs some attention, but conventional filtering schemes are expected to be sufficient to allow stable solutions.

If the amplitudes of the disturbances are high and particularly if helical waves of opposite azimuthal wavenumber are included, there is an efficient oblique mode breakdown to turbulence and the above method will eventually include sound generated from this process, in addition to the large structure sound. Typically this occurs further downstream where the jet centreline velocity has reduced, giving less explosive breakdown than if the breakdown occurs near the end of the potential core. Provided the filtering is effective this need not limit the above method with respect to large structure sound, but it can lead to a situation where the calculations include sound due to both large and small structures. This actually seems to give quite realistic acoustic spectra (at least up to St=1) compared to experiment [8], but it does mean that the large and small structure sound become combined. Another drawback for high amplitude forcing is that the resolved Reynolds stresses cause the mean flow to depart from the prescribed base flow, which is inconsistent.

6.3. Perturbation evolution from Parabolised Stability Equations

A much cheaper approach can be constructed using stability analysis to generate the hydrodynamic response and then adding source terms into a linear Euler code to compute the acoustic field. The stability analysis can be linear or nonlinear, leading to some subtleties. The linear stability approach was investigated by [21], using the linear Parabolised Stability Equations (PSE). For a given base flow (which was specified analytically, but could equally well have come from a RANS calculation), the Orr-Sommerfeld equation was solved at a location just downstream of the nozzle exit. Eigenmodes were stored over a range of frequencies and for each frequency a linear PSE calculation was run to find the downstream response. The PSE includes nonparallel effects at first order and was therefore preferred to the even simpler Orr-Sommerfeld approach. The PSE solutions were stored and acoustic source terms calculated for all relevant quadratic interactions, which were fed into a wave equation solver. Results were quite realistic for the most unstable wave combinations. A subtle point concerned the interaction of waves with very close frequencies which were found to give very high (certainly unphysical) levels of sound radiations. This issue was explored in [8] where DNS of the same cases showed that what actually happens is that the difference mode for two close frequencies loses energy to its immediate harmonics, which explains why it doesn't radiate as efficiently as was seen in the linear PSE model. The solution to this issue is to move to nonlinear PSE, which captures such interactions. Nonlinear PSE are currently being evaluated against DNS [18]. It is already clear from this work that the radial growth rate of jets is such that a perfect agreement with DNS is not to be expected, since PSE only includes nonparallel effects to first order. Nevertheless the method appears to be feasible and the fact that it is significantly cheaper than the simulation-based approach means that it is worth further study.

7. Conclusions

In this paper we have provided substantial evidence that quadratic nonlinear interactions of instability waves contain the correct physical mechanisms to understand the component of sound from subsonic jets that is commonly labelled 'large structure' sound. It should be noted that this goes much further than just identifying sound as originating in wavepackets, or as Mach waves. In particular the sound radiation from any forced interactions can be computed, giving insight into which interactions are responsible for which components of jet noise. Additionally, the simulations can be extended to include breakdown to small scales, which separates out the 'small structure' sound. In the latter part of the paper have provided some more speculative remarks on how this insight can be extended to predictive schemes based on currently available tools, including RANS-CFD, linear and nonlinear instability theory and linearized Euler equations. The combination seems to offer the potential to connect upstream nozzle flow control with the radiated sound. Some areas needing further research have been identified, in particular the range of applicability of linear and nonlinear methods based on the parabolized stability equations, as compared to a truncated Navier-Stokes approach.

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